# Blacksburg Math Circle

## Problems on Last Digits and Remainders 11/5/2016

1. What is the last digit of $777^{777}$?
2. What is the last digit of $2^{1999} + 3^{2000}$?
3. Find the remainder when 5 is divided into $19^{2005} - 21^{23}$.
4. Find the last 4 digits of $2016^{2016}$.
5. Find the remainder when the number $3^{1989}$ is divided by 7.
6. Prove that $2222^{5555} + 5555^{2222}$ is divisible by 7.
7. If it is known that $a + 1$ is divisible by 3, prove that $4 + 7a$ is divisible by 3
8. It is known that $2 + a$ and $35 - b$ are divisible by 11. Prove that $a + b$ is divisible by 11.
9. Show that 3 divides a number if and only if it divides the sum of its digits.
10. Prove that $n^5 + 4n$ is divisible by 5 for any integer $n$.
11. Prove that $n^3 - n$ is divisible by 6 for any integer $n$.
12. Prove that $n^3 - n$ is divisible by 24 for any odd $n$.
13. Given natural numbers $a$, $b$ and $c$ such that $a + b + c$ is divisible by 6, prove that $a^3 + b^3 + c^3$ is also divisible by 6.